CrossFit Open Participation Analysis

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1 The Model

Numerical analysis suggests the logarithm of the fractional (percentage) increase to be a straight line:

$$\ln\left(\frac{f(t+1)}{f(t)} - 1\right) = at + b \tag{1}$$

where t is the year since the beginning of the CrossFit Open and $f(t)$ is the number of participants. Coalescing the argument of $\ln$ and exponentiating both sides yields:

$$\frac{f(t+1) - f(t)}{f(t)} = e^{at+b} \tag{2}$$

At this point it is necessary to assume that the growth rate's dependence on time is the same for fractional years as it is for full years. That is, even though the Open is not held two, three, or twelve times per year, we assume that it is and that the logarithm of the growth still follows a straight line:

$$\frac{f(t+\delta) - f(t)}{f(t)} = \delta e^{at+b} \tag{3}$$

where $\delta \in [0, 1]$. The equation reduces to its original form if $\delta = 1$, and the growth rate for $\delta = 0$ is obviously zero. Dividing both sides by δ:

$$\frac{f(t+\delta) - f(t)}{\delta f(t)} = e^{at+b} \tag{4}$$

In the limit of arbitrary small δ, a derivative of $f(t)$ emerges:

$$\lim_{\delta \to 0} \frac{f(t+\delta) - f(t)}{\delta} \frac{1}{f(t)} = e^{at+b} \tag{5}$$

$$\frac{f'(t)}{f(t)} = e^{at+b} \tag{6}$$

$$\partial_t \ln f(t) = e^{at+b} \tag{7}$$

for ease of calculation, $g(t) \equiv \ln f(t)$ is introduced

$$g'(t) = e^{at+b} \tag{8}$$

$$g(t) = \frac{1}{a} e^{at+b} + c \tag{9}$$

Substituting $g(t)$ yields the model for CrossFit Open participation as a function of time:

$$f(t) = Ce^{\frac{1}{a} e^{at+b}} \tag{10}$$

which can alternatively be written as:

$$f(t) = Ce^{\beta e^{\alpha t}} \tag{11}$$